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Supergravity solutions for localised brane intersections

Arvind Rajaraman

Department of Physics and Astronomy, Rutgers University
Piscataway, NJ 08855, Usa
E-mail: arvindra@muon.rutgers.edu

Abstract: We present a general method for constructing supergravity solutions for intersecting branes. The solutions are written in terms of a single function, which is the solution to a nonlinear differential equation. We illustrate this procedure in detail for the case of M2-branes ending on M5-branes. We also present supergravity solutions for strings ending on Dp-branes. Unlike previous results in the literature, these branes are completely localized.

Keywords: D-branes, Classical Theories of Gravity, Supergravity Models.

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Contents

1. Introduction

Supergravity solutions representing D-branes, NS 5-branes, and M-branes have been known for a long time now [1]. It is also known that strings can end on D-branes [2] and branes can end on other branes [3]. However, supergravity solutions representing such intersecting objects are, by and large, unknown.

Previous work in the literature has mostly focused on cases where one of the branes is delocalized in some direction [4]. There are only a few cases where all the branes have been localized [5]–[10], but these appear to be found on a case by case basis.

In this paper, we will provide a completely general method for finding supergravity solutions representing intersecting branes. This is based on the method introduced by [9] (See also [10]). Using this approach, which we describe in more detail below, we are able to write the solution in terms of the solution to a single nonlinear differential equation. Unfortunately, we have been unable to find explicit solutions of this differential equation. Nevertheless, we regard this as a solution in principle, and it is far simpler than attempting to solve the coupled Maxwell-Einstein equations.

The idea in this approach is to use the supersymmetry equations (the equations for conserved Killing spinors) as linear equations determining the field strengths in terms of the metric. This enables us to write all the fields in terms of the parameters of the metric.

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It is then straightforward to substitute these expressions into the equations of motion. By the miracle of supersymmetry, we find that all the equations of motion simplify into a few simple equations. All the parameters can then be determined in terms of one function, which is in turn a solution to a nonlinear differential equation. Although we have been unable to find explicit solutions to this nonlinear equation, nevertheless, we emphasize that the solution is completely determined in principle by this differential equation.

We carry out these steps in great detail for the case of M2-branes ending on M5-branes. This method can be easily generalized to all other systems of intersecting branes in M-theory and string theory, and as a further example, we present solutions for strings ending on Dp-branes. Finally, we list several open problems.

2. M2-branes ending on M5-branes

2.1 Setup

The notation will be as follows. Indices with a tilde over them will denote curved (world) indices, and indices without a tilde will be tangent indices.

The M5-brane is oriented along the directions x_0 (time), x_1, x_2, x_3, x_4, x_5. The M2-brane is oriented along the directions x_0, x_1, x_6. Indices $(2,3,4,5)$ will be labeled by $a, b, \ldots$, and indices $(7,8,9,10)$ will be labeled $\alpha, \beta, \ldots$. (We will unfortunately have some situations below where we will use the number 10 as a subscript e.g. F_{8910a}. The correct reading should be clear from the context.)

The M5-brane and M2-brane act as sources for $a^{(3)}_{\alpha\beta\gamma}$ and $a^{(3)}_{016}$ respectively. Correspondingly, we expect the field strengths $f_{\alpha\beta\gamma\delta}, f_{\alpha\beta\gamma a}, f_{\alpha\beta\gamma 6}$, and $f_{01a6}, f_{016\alpha}$ to be nonzero. Furthermore, the M2 brane acts as a source for the $U(1)$ field on the worldvolume of the M5-brane. If the worldvolume field strength is denoted by h, then h_{01a} and h_{abc} are nonzero. This field acts as a source for $a^{(3)}_{abc}$ and $a^{(3)}_{01a}$. Correspondingly, we also expect $f_{01a\alpha}$, and $f_{2345}, f_{abc6}, f_{abc\alpha}$ to be nonzero.

Also, since $\partial_a X^6$ is nonzero on the M5 brane worldvolume, we expect g_{a6} to be nonzero.

The nonzero components of the field strengths will be denoted

$$F_{\alpha\beta\gamma\delta}\,, \quad F_{\alpha\beta\gamma a}\,, \quad F_{\alpha\beta\gamma 6}\,; \quad G_{01a6}\,, \quad G_{01a\alpha}\,, \quad G_{016\alpha}\,; \quad H_{2345}\,, \quad H_{abc6}\,, \quad H_{abc\alpha}\,. \tag{2.1}$$

We have chosen a notation where different components of the field strength are denoted by F, G, or H depending on the indices.

There is a symmetry of rotations in the (x_α) directions. Due to this symmetry, we can reduce the number of independent components of the field strength we have to determine. For instance, if we find F_{8910a}, we can immediately find the field stregth components F_{7910a} by the rotation (which here gives $F_{7910a} = \frac{-x_8}{x_7} F_{8910a}$). This symmetry will be assumed henceforth.

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2.2 The equations for supersymmetry

We can now analyse the equation for conserved supercharges

$$\delta\psi_{\tilde{\mu}} = \partial_{\tilde{\mu}}\epsilon - \frac{1}{4}\omega^{ab}_{\tilde{\mu}}\gamma_{ab} + \frac{i}{288}(\gamma_{\tilde{\mu}}^{\alpha\beta\gamma\delta} - 8\delta^{\alpha}_{\tilde{\mu}}\gamma^{\beta\gamma\delta})f_{\alpha\beta\gamma\delta}\epsilon = 0\,. \tag{2.2}$$

These equations are rather cumbersome. For example, for $\mu = 1$, we have

$$\begin{aligned}\left(\frac{1}{2}\omega_1^{1a}\gamma_a + \frac{1}{2}\omega_1^{16}\gamma_6 + \frac{1}{2}\omega_1^{1\alpha}\gamma_\alpha\right)\epsilon - \frac{i}{12}F_{78910}\gamma^{78910}\epsilon - \frac{i}{12}H_{2345}\gamma^{2345}\epsilon- \\ -\frac{i}{12}\left(F_{789a}\gamma^{789a} + F_{7810a}\gamma^{7810a} + F_{7910a}\gamma^{7910a} + F_{8910a}\gamma^{8910a}\right)\epsilon- \\ -\frac{i}{12}\left(F_{7896}\gamma^{7896} + F_{78106}\gamma^{78106} + F_{79106}\gamma^{79106} + F_{89106}\gamma^{89106}\right)\epsilon- \\ -\frac{i}{12}\left(H_{234\alpha}\gamma^{234\alpha} + H_{235\alpha}\gamma^{235\alpha} + H_{245\alpha}\gamma^{245\alpha} + H_{345\alpha}\gamma^{345\alpha}\right)\epsilon- \\ -\frac{i}{12}\left(H_{2346}\gamma^{2346} + H_{2356}\gamma^{2356} + H_{2456}\gamma^{2456} + H_{3456}\gamma^{3456}\right)\epsilon+ \\ +\frac{i}{6}\left(G_{01a\alpha}\gamma^{01a\alpha} + G_{01a6}\gamma^{01a6} + G_{016\alpha}\gamma^{016\alpha}\right)\epsilon = 0\,.\end{aligned} \tag{2.3}$$

All indices above are tangent space indices; we have defined $\omega_a^{bc} = e_a^{\tilde{\mu}}\omega_{\tilde{\mu}}^{bc}$.

To simplify the problem, we note that since we are looking for BPS solutions, we expect to be able to superpose them. In this case, we should be able to superpose M2 branes with arbitrary coordinates in the directions x_a. We can therefore separate the SUSY equations into terms which are even in x_a, and terms which are odd in x_a. These equations should be separately satisfied. The terms involving a ∂_a clearly produces a term odd in x_a, while $\partial_6, \partial_\alpha$ produce terms even in x_a.

In addition, we will take the ansatz for the spinor to be

$$\epsilon = (g_{11})^{1/4}\epsilon_0\,, \tag{2.4}$$

where ϵ_0 is a constant spinor. This ansatz is justified in [11].

The parts of the equations odd in x_4 are

$$\begin{aligned}\frac{1}{2}\omega_1^{14}\gamma_4\epsilon + \left(-\frac{i}{12}H_{235\alpha}\gamma^{235\alpha} - \frac{i}{12}H_{2356}\gamma^{2356} + \frac{i}{6}G_{014\alpha}\gamma^{014\alpha} + \frac{i}{6}G_{0146}\gamma^{0146}\right)\epsilon- \\ -\frac{i}{12}\left(F_{7894}\gamma^{7894} + F_{78104}\gamma^{78104} + F_{79104}\gamma^{79104} + F_{89104}\gamma^{89104}\right)\epsilon = 0\end{aligned}$$

$$\begin{aligned}\frac{1}{2}(\omega_5^{54} - \omega_1^{14})\gamma_4\epsilon+ \\ +\left(\frac{i}{4}H_{235\alpha}\gamma^{235\alpha} + \frac{i}{4}H_{2356}\gamma^{2356} - \frac{i}{4}G_{014\alpha}\gamma^{014\alpha} - \frac{i}{4}G_{0146}\gamma^{0146}\right)\epsilon = 0\end{aligned}$$

$$\frac{1}{2}\left(\omega_5^{54}\gamma_4 + \omega_4^{67}\gamma_{467}\epsilon\right) + \left(\frac{i}{4}H_{235\alpha}\gamma^{235\alpha} + \frac{i}{4}H_{2356}\gamma^{2356}\right)\epsilon\times$$

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$$\times\left(-\frac{i}{4}G_{014\alpha}\gamma^{014\alpha}-\frac{i}{4}G_{0146}\gamma^{0146}-\frac{i}{4}F_{89104}\gamma^{89104}\right)\epsilon=0$$
$$\frac{1}{2}(\omega_6^{64}-\omega_1^{14})\gamma_4\epsilon-\frac{1}{2}\omega_6^{4\alpha}\gamma_{64\alpha}\epsilon+\left(\frac{i}{4}H_{2356}\gamma^{2356}-\frac{i}{4}G_{014\alpha}\gamma^{014\alpha}\right)\epsilon=0$$
$$-\frac{1}{2}\omega_7^{64}\gamma_{764}+\left(\frac{i}{4}G_{0147}\gamma^{0147}+\frac{i}{4}H_{2357}\gamma^{2357}-\frac{i}{4}F_{89104}\gamma^{89104}\right)\epsilon=0\,. \tag{2.5}$$

The parts of the equations even in all x_a are

$$\frac{1}{2}\omega_1^{16}\gamma_6\epsilon+\frac{1}{2}\omega_1^{1\alpha}\gamma_\alpha\epsilon+\left(\frac{i}{6}G_{016\alpha}\gamma^{016\alpha}-\frac{i}{12}H_{2345}\gamma^{2345}-\frac{i}{12}F_{78910}\gamma^{78910}\right)\epsilon-$$
$$-\frac{i}{12}\left(F_{7896}\gamma^{7896}+F_{78106}\gamma^{78106}+F_{79106}\gamma^{79106}+F_{89106}\gamma^{89106}\right)\epsilon=0$$
$$\frac{1}{2}(\omega_5^{56}-\omega_1^{16})\gamma_6\epsilon+\frac{1}{2}(\omega_5^{5\alpha}-\omega_1^{1\alpha})\gamma_\alpha\epsilon+\left(\frac{i}{4}H_{2345}\gamma^{2345}-\frac{i}{6}G_{016\alpha}\gamma^{016\alpha}\right)\epsilon=0$$
$$\frac{1}{2}(\omega_6^{6\alpha}-\omega_1^{1\alpha})\gamma_\alpha\epsilon+\frac{i}{4}\left(F_{7896}\gamma^{7896}+F_{78106}\gamma^{78106}+F_{79106}\gamma^{79106}+F_{89106}\gamma^{89106}\right)\epsilon=0$$
$$\frac{1}{2}(\omega_1^{17}-\omega_8^{87})\gamma_7\epsilon+\frac{i}{4}G_{0167}\gamma^{0167}\epsilon-\frac{i}{4}F_{89106}\gamma^{89106}\epsilon=0\,. \tag{2.6}$$

These equations should preserve one quarter of the supersymmetries. In other words, each equation should be proportional to a linear combination of $P_1\epsilon_0$ and $P_2\epsilon_0$, where $P_1\epsilon_0=P_2\epsilon_0=0$ and P_1, P_2 are projection operators.

A glance at the equations (2.5) and (2.6) shows that the only possible choice of projection operators is (upto signs)

$$\begin{aligned}P_1\epsilon_0 &\equiv (1+i\gamma^{678910})\epsilon_0=0\\ P_2\epsilon_0 &\equiv (1+i\gamma^{016})\epsilon_0=0\,.\end{aligned} \tag{2.7}$$

With these projection operators, we can reduce the matrix equations (2.5) and (2.6) to a set of algebraic equations. We will treat these algebraic equations as equations determinining the various field strengths as functions of the spin connections (and thereby implicitly as functions of the metric.)

The equations for the field strengths are then

$$\begin{aligned}G_{0164} &= 4\omega_1^{14}+2\omega_5^{54}\\ G_{0167} &= 2\omega_1^{17}-2\omega_4^{47}\\ G_{0147} &= 2\omega_4^{67}\\ H_{2356} &= 4\omega_5^{54}+2\omega_1^{14}\\ H_{2357} &= -2\omega_4^{67}\end{aligned}$$

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$$
\begin{aligned}
H_{2345} &= 2\omega_4^{46} - 2\omega_1^{16} \\
F_{48910} &= -2\omega_4^{67} \\
F_{78910} &= -2\omega_4^{46} - \omega_1^{16} \\
F_{68910} &= 4\omega_4^{47} + 2\omega_1^{17} \,.
\end{aligned}
\tag{2.8}
$$

In addition, we get the constraints

$$
\begin{aligned}
\omega_1^{16} + \omega_4^{46} + \omega_7^{76} &= 0 \\
\omega_1^{18} + \omega_4^{48} + \omega_7^{78} &= 0 \\
\omega_1^{14} + \omega_5^{54} + \omega_7^{74} &= 0 \\
\omega_4^{67} &= \omega_6^{47} = -\omega_7^{64} \\
\omega_6^{67} + 2\omega_4^{47} &= 0 \\
\omega_6^{64} &= 2\omega_5^{54} + 2\omega_1^{14} \,.
\end{aligned}
\tag{2.9}
$$

In all the equations above, all the indices are tangent space indices; we have defined $\omega_c^{ab} = e_c^{\tilde{\mu}} \omega_{\tilde{\mu}}^{ab}$.

We can solve all the constraints above by the metric ansatz

$$
\begin{aligned}
e_{0\tilde{0}} &= e_{1\tilde{1}} = \lambda^{-1/3} H^{-1/6} \\
e_{2\tilde{2}} &= e_{3\tilde{3}} = e_{4\tilde{4}} = e_{5\tilde{5}} = \lambda^{1/6} H^{-1/6} \\
e_{6\tilde{6}} &= \lambda^{-1/3} H^{1/3} \\
e_{7\tilde{7}} &= e_{8\tilde{8}} = e_{9\tilde{9}} = e_{10\tilde{10}} = \lambda^{1/6} H^{1/3} \\
e_{6\tilde{a}} &= \phi_a e_{6\tilde{6}}
\end{aligned}
\tag{2.10}
$$

with the constraint

$$
\partial_6 (H \phi_a) = \partial_a H \,. \tag{2.11}
$$

2.3 The equations of motion

We now look at the equations of motion. Away from the M5-brane, we can look at the vacuum equations of motion, which for the gauge fields is

$$
\partial_\mu f^{\mu\nu\rho\sigma} = \frac{1}{2.(24)^2} \epsilon^{\nu\rho\sigma\tilde{a}\tilde{b}\tilde{c}\tilde{d}\tilde{e}\tilde{f}\tilde{g}\tilde{h}} f_{\tilde{a}\tilde{b}\tilde{c}\tilde{d}} f_{\tilde{e}\tilde{f}\tilde{g}\tilde{h}} \,. \tag{2.12}
$$

We can now substitute the field strengths found above into these equations. Remarkably, all the equations of motion and the Bianchi identities collapse to the equations

$$
\partial_a \left(\phi_a\right) - \frac{1}{H} \partial_6 \lambda - \frac{1}{2} \partial_6 \phi_a^2 = 0 \tag{2.13}
$$

and

$$
\partial_\alpha^2 \left(H\right) + \partial_6^2 (H\lambda) = 0 \,. \tag{2.14}
$$

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When we include sources, we need to modify the second equation. Defining

$$\partial_\alpha^2 (H) + \partial_6^2(H\lambda) = \partial_6 Q \tag{2.15}$$

we have

$$\begin{aligned}
\partial_7 F_{a8910} - \partial_8 F_{a7910} + \partial_9 F_{a7810} - \partial_{10} F_{a789} - \partial_a F_{78910} &= \partial_a Q \\
\partial_\mu G^{\mu 01a} + \frac{1}{2.(24)^2}\epsilon^{\tilde{0}\tilde{1}\tilde{a}\tilde{A}\tilde{B}\tilde{C}\tilde{D}\tilde{E}\tilde{F}\tilde{G}\tilde{H}} f_{\tilde{A}\tilde{B}\tilde{C}\tilde{D}} f_{\tilde{E}\tilde{F}\tilde{G}\tilde{H}} &= \partial_a Q - i\phi\partial_6 Q \\
\partial_6\left(\partial_\mu G^{\mu 016} + \frac{1}{2.(24)^2}\epsilon^{\tilde{0}\tilde{1}\tilde{6}\tilde{A}\tilde{B}\tilde{C}\tilde{D}\tilde{E}\tilde{F}\tilde{G}\tilde{H}} f_{\tilde{A}\tilde{B}\tilde{C}\tilde{D}} f_{\tilde{E}\tilde{F}\tilde{G}\tilde{H}}\right) &= -\partial_a^2 Q
\end{aligned} \tag{2.16}$$

Q thus parametrizes the perturbation of the original M5-brane due to the addition of the M2-brane.

2.4 Linearized analysis

At the linearized level, we can drop the terms involving ϕ. If we then take

$$\begin{aligned}
\partial_\mu G^{\mu\tilde{0}\tilde{1}\tilde{a}} + \frac{1}{2.(24)^2}\epsilon^{\tilde{0}\tilde{1}\tilde{a}\tilde{A}\tilde{B}\tilde{C}\tilde{D}\tilde{E}\tilde{F}\tilde{G}\tilde{H}} f_{\tilde{A}\tilde{B}\tilde{C}\tilde{D}} f_{\tilde{E}\tilde{F}\tilde{G}\tilde{H}} &= \delta(x_\alpha)\delta(x_6)h^{01a} \\
\partial_\mu G^{\mu\tilde{0}\tilde{1}\tilde{6}} + \frac{1}{2.(24)^2}\epsilon^{\tilde{0}\tilde{1}\tilde{6}\tilde{A}\tilde{B}\tilde{C}\tilde{D}\tilde{E}\tilde{F}\tilde{G}\tilde{H}} f_{\tilde{A}\tilde{B}\tilde{C}\tilde{D}} f_{\tilde{E}\tilde{F}\tilde{G}\tilde{H}} &= \begin{cases} \delta^4(x_a)\delta^4(x_\alpha)\,, & x_6 \geq 0\,; \\ 0\,, & \text{else}\,. \end{cases}
\end{aligned} \tag{2.17}$$

the above equations are satisfied provided

$$\partial_a h^{01a} = \delta(x_a),. \tag{2.18}$$

In other words, we have a gauge field strength on the M5-brane corresponding to the field of an electric charge on a line at $x_a = 0$. Also, we have a membrane (the source for A_{016}) extending along the positive x_6 axis. This is exactly the configuration of a M2-brane ending on an M5-brane.

At the linearized level, the equations (2.11), (2.13) and (2.15) also simplify to

$$\begin{aligned}
\partial_6(H\phi_a) &= \partial_a H \\
\partial_a(H\phi_a) &= \partial_6\lambda \\
\partial_\alpha^2(H\phi_a) + \partial_a\partial_6(H\lambda) &= \partial_a Q
\end{aligned} \tag{2.19}$$

which can be simplified to the linear equation

$$H\partial_a^2\lambda + \partial_6^2\lambda + \partial_\alpha^2\lambda = \frac{1}{\partial_6}\partial_a^2 Q = \begin{cases} \delta^4(x_a)\delta^4(x_\alpha)\,, & x_6 \geq 0\,; \\ 0\,, & \text{else}\,. \end{cases} \tag{2.20}$$

More generally, we have multiple M2 branes ending at different points on the M5-brane. The general solution, at the linear level, is then

$$\lambda = \int dx' K(x,x')\rho(x')\,, \tag{2.21}$$

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where we have introduced the Green's function K, satisfying

$$(H_0\partial_a^2 + \partial_6^2 + \partial_\alpha^2)K(x,x') = \delta(x-x') \tag{2.22}$$

H_0 is the harmonic function describing the M5-branes without the M2-branes and $\rho(x')$ is the membrane source density.

The full nonlinear solution no longer has pointlike sources. The source Q is now a nontrivial function of x_a and x_6, and even the identification of Q as a worldvolume field is a little problematic.

However, at long distances from the M2-branes, the linearized analysis still applies, since λ is small. Also, very close to any M2-brane, where λ diverges, the solution (2.21) should still apply, since the effects of the membrane dominate.

At the nonlinear level, we can then say that λ is a solution to the equations (2.11), (2.13) and (2.15), subject to the condition that

$$\lambda = \int dx' K(x,x')\rho(x') \tag{2.23}$$

both when λ is small or when it is very large. This is expected to completely specify the solution, although we cannot explicitly solve the equations.

2.5 Comments

1. One might ask why one needs to look at the equations of motions at all. After all, once we impose the constraint (2.11), we have a solution that is supersymmetric. It is well known that a solution preserving a supersymmetry should also satisfy the equations of motion.

The point is that imposing the supersymmetry constraints does not completely specify the sources. The M2-branes along x_0, x_1, x_6, and the M5-branes along $x_0, x_1, x_2, x_3, x_4, x_5$, preserve the supersymmetries (2.7). But one can also add M5-branes oriented along $x_0, x_1, x_7, x_8, x_9, x_{10}$ without breaking any more supersymmetries. The equation (2.13) precisely sets the number of these M5-branes to zero, so that we have the sources we started with.

2. The second equation (2.15) determines the positions of the M5-brane and M2-brane sources.

One expects on physical grounds that the M5-branes should not deform in the x_α directions. In this case, one should find that $Q = \delta^4(x_\alpha)Q_1(x_a, x_6)$.

It is however not clear how to show that the differential equations (2.11), (2.13), (2.15) are consistent with this expectation. One could, in principle, worry that when one tried to integrate in the equations from $x_\alpha = \infty$, one gets a singularity away from the origin. The solution would then be valid only outside this region (somewhat like the enhancon phenomenon [12]). It would be extremely surprising (at least to this author) if such a phenomenon should occur in this system, so we shall assume that in fact, $Q = \delta^4(x_\alpha)Q_1(x_a, x_6)$.

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2.6 Summary

The solution for M2-branes ending on M5-branes is given by the metric (2.10)

$$\begin{aligned} ds^2 = &\lambda^{-2/3}H^{-1/3}(-dx_0^2+dx_1^2)+\lambda^{1/3}H^{-1/3}(dx_2^2+dx_3^2+dx_4^2+dx_5^2)+ \\ &+\lambda^{1/3}H^{2/3}(dx_7^2+dx_8^2+dx_9^2+dx_{10}^2)+\lambda^{-2/3}H^{2/3}(dx_6+\phi_a dx_a)^2 . \end{aligned} \tag{2.24}$$

The field strengths are given by

$$\begin{aligned} G_{016a} &= 4\omega_1^{1a}+2\omega_5^{5a} \\ G_{016\alpha} &= 2\omega_1^{1\alpha}-2\omega_4^{4\alpha} \\ G_{01a\alpha} &= 2\omega_a^{6\alpha} \\ H_{abc6} &= (-\epsilon^{abcd})(4\omega_5^{5d}+2\omega_1^{1d}) \\ H_{abc\alpha} &= 2\epsilon^{abcd}\omega_d^{6\alpha} \\ H_{2345} &= 2\omega_4^{46}-2\omega_1^{16} \\ F_{\alpha\beta\gamma\delta} &= -2\epsilon^{\alpha\beta\gamma\delta}\omega_a^{6\alpha} \\ F_{78910} &= -2\omega_4^{46}-\omega_1^{16} \\ F_{6\beta\gamma\delta} &= \epsilon^{\alpha\beta\gamma\delta}(4\omega_4^{4\alpha}+2\omega_1^{1\alpha}) . \end{aligned} \tag{2.25}$$

We also have the equations (2.11), (2.13), and (2.15) which determine the parameters

$$\begin{aligned} \partial_6(H\phi_a) &= \partial_a H \\ \partial_a(\phi_a)-\frac{1}{H}\partial_6\lambda-\frac{1}{2}\partial_6\phi_a^2 &= 0 \\ \partial_\alpha^2(H)+\partial_6^2(H\lambda) &= \delta^4(x_\alpha)\partial_6 Q , \end{aligned} \tag{2.26}$$

where Q is a function of x_a, x_6.

All the equations of motion and all the Bianchi identities are solved by the above ansatz.

Far away from the M2-branes, we have the boundary condition

$$H \to H_0 \qquad \phi_a \to 0 \qquad \lambda \to 1 . \tag{2.27}$$

The first linear perturbation in λ is

$$\lambda = \int dx' K(x,x')\rho(x') + 1 , \tag{2.28}$$

where $\rho(x')$ is the membrane source density, and $K(x,x')$ is the Green's function satisfying

$$(H_0\partial_a^2+\partial_6^2+\partial_\alpha^2)K(x,x') = \delta(x-x') , \tag{2.29}$$

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where H_0 is the harmonic function describing the M5-brane background without the M2-branes. Furthermore, λ is expected to behave as the above equation (2.28) in regions where λ diverges.

The system of equations (2.11), (2.13), and (2.15) can further be simplified to the following equations

$$\begin{aligned} H\phi_a &= \partial_a\partial_6\tau \\ H &= \partial_6^2\tau \\ \lambda &= \partial_a^2\tau - H\phi_a^2\,, \end{aligned} \tag{2.30}$$

where the function τ satisfies the nonlinear differential equation

$$\partial_\alpha^2\tau + \partial_6^2\tau\partial_a^2\tau - (\partial_a\partial_6\tau)^2 = \delta^4(x_\alpha)\frac{1}{\partial_6}Q\,. \tag{2.31}$$

3. Strings ending on Dp-branes

We can carry out a very similar analysis for strings ending on Dp-branes. We shall take the Dp-branes oriented along the directions $x_0 \ldots x_p$, the string is oriented along x_0, x_9. We shall denote the directions $x_1 \ldots x_p$ collectively as x_a, and the directions $x_{p+1} \ldots x_8$ collectively as x_α.

The Killing spinors preserved by this configuration are given by $\epsilon = (g_{00}^{1/4})\epsilon_0$, where ϵ_0 is a constant spinor satisfying

$$\begin{aligned} \gamma^{09}\epsilon_0 &= \eta_1\epsilon_0 \\ \gamma^{01\ldots p}\epsilon_0 &= \eta_2\epsilon_0\,, \end{aligned} \tag{3.1}$$

where η_1, η_2 are constants.

The Killing equations can be written in the generic form

$$\begin{aligned} &\omega_\mu^{ab}\gamma_{ab}\epsilon + (a_G G_{\mu ab}\gamma^{ab} + b_G G_{abc}\gamma_\mu^{abc})\epsilon + \\ &+(a_H H_{\mu a_1\ldots a_{p-2}}\gamma^{a_1\ldots a_{p-2}} + b_H H_{a_1\ldots a_{p-1}}\gamma_\mu^{a_1\ldots a_{p-1}})\epsilon + \\ &+(a_F F_{\mu a_1\ldots a_p}\gamma^{a_1\ldots a_p} + b_F F_{a_1\ldots a_p}\gamma_\mu^{a_1\ldots a_p})\epsilon = 0 \end{aligned} \tag{3.2}$$

We require the Killing spinor equations to be satisfied after the imposition of the projections (3.1). The Killing spinor equations then yield a large set of algebraic equations and constraints similar to (2.8) and (2.9). We can then proceed to solve these constraints.

The results of the analysis are presented directly. The solution presented is supersymmetric and all the equations of motion and Bianchi identities are satisfied.

The metric components are found by noting that the diagonal components are the products of the corresponding component in the metric of the string and the

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p-brane metric (i.e. $g_{ii} = g_{ii}^{(\mathrm{string})} g_{ii}^{(p-\mathrm{brane})}$.) (We will work in the string frame)

$$\begin{aligned}
e_{0\tilde{0}} &= \lambda^{-1/2} H^{-1/4} \\
e_{1\tilde{1}} &= e_{2\tilde{2}} = \ldots = e_{p\tilde{p}} = H^{-1/4} \\
e_{9\tilde{9}} &= \lambda^{-1/2} H^{1/4} \\
e_{(p+1)(\tilde{p+1})} &= \ldots = e_{8\tilde{8}} = H^{1/4} \,.
\end{aligned} \tag{3.3}$$

In addition, we have an off diagonal component.

$$e_{9\tilde{a}} = \phi_a e_{9\tilde{9}} \,. \tag{3.4}$$

The dilaton is

$$e^{\phi} = \lambda^{-1/2} H^{-\frac{p-3}{4}} \,. \tag{3.5}$$

The equations for the field strengths are then (all indices are tangent space indices)

$$\begin{aligned}
(a_G \eta_1) G_{0a9} &= -\omega_2^{2a} - \omega_0^{0a} \\
(a_G \eta_1) G_{09\alpha} &= \omega_0^{0\alpha} - \omega_2^{2\alpha} \\
(a_G \eta_1) G_{0a\alpha} &= \omega_\alpha^{a9} \\
(a_H \eta_1 \eta_2) e^{\phi} \epsilon^{a_1 a_2 \ldots a_p} H_{9 a_2 \ldots a_p} &= -\omega_2^{2 a_1} \\
(a_H \eta_1 \eta_2) e^{\phi} \epsilon^{a_1 a_2 \ldots a_p} H_{\alpha a_2 \ldots a_p} &= \frac{1}{2} \omega_\alpha^{a_1 9} \\
(a_H \eta_1 \eta_2) e^{\phi} H_{1 \ldots p} &= \frac{1}{2} (\omega_2^{29} - \omega_0^{09}) \\
(a_F \eta_2) e^{\phi} F_{\alpha 01 \ldots p} &= \omega_2^{2\alpha} \\
(a_F \eta_2) e^{\phi} \epsilon^{a_1 a_2 \ldots a_p} F_{\alpha 90 a_2 \ldots a_p} &= -\omega_\alpha^{a_1 9} \\
(a_F \eta_2) e^{\phi} F_{901 \ldots p} &= \frac{1}{2} (\omega_0^{09} + \omega_2^{29}) \,.
\end{aligned} \tag{3.6}$$

The equations relating the various parameters are

$$\begin{aligned}
\partial_9 (H \phi_a) &= \partial_a H \\
H^{-1} \partial_9 \lambda + \frac{1}{2} \partial_9 (\phi_a^2) &= \partial_a \phi_a \\
\partial_9^2 (H \lambda) + \partial_\alpha^2 (H) &= \delta(x_\alpha) \partial_9 Q
\end{aligned} \tag{3.7}$$

which can be simplified in terms of a single function τ

$$\begin{aligned}
H &= \partial_9^2 \tau \\
H \phi_a &= \partial_a \partial_9 \tau \\
\lambda &= \partial_a^2 \tau - H \phi_a^2 \,,
\end{aligned} \tag{3.8}$$

where τ satisfies

$$\partial_\alpha^2 \tau + \partial_9^2 \tau \partial_a^2 \tau - (\partial_a \partial_6 \tau)^2 = \delta(x_\alpha) \frac{1}{\partial_9} Q \tag{3.9}$$

which is the same differential equation as in the previous section.

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4. Open questions

We have reduced the construction of intersecting brane systems to solving a single nonlinear partial differential equation. Much of the physics is contained in the solution to this equation, which, unfortunately, we have been unable to find exactly. It would be useful to be able to extract some information from the equation, for instance, to know whether it admits multiple solutions in some cases.

There are several potential applications of these solutions. By the AdS/CFT correspondence [13], some of these solutions can be mapped to configurations in strongly coupled gauge theories. For example, the solution for k D-strings ending on N D3-brane is dual to a configuration of k monopoles in SU(N) $\mathcal{N}=4$ Yang-Mills theory. We can then extract quantities like the quark monopole potential etc. in this limit.

Similarly, these configurations, can be used to construct supergravity duals for any gauge theory that can be engineered by putting branes on orbifolds. These include pure $\mathcal{N}=2$ gauge theories and $\mathcal{N}=1$ gauge theories. One can also study solitons in these theories.

One very interesting open question is to find the near extremal solutions. It is, of course, not possible to use the supersymmetry equations there, but the solutions found here should be a useful starting point to get the nonextremal solutions. Such solutions would give us information about the thermodynamics of theories with less supersymmetry.

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References

[1] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys.* **B 360** (1991) 197.

[2] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [`hep-th/9510017`].

[3] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [`hep-th/9512059`].

[4] D. Youm, *Partially localized intersecting BPS branes*, *Nucl. Phys.* **B 556** (1999) 222.

[5] N. Itzhaki, A.A. Tseytlin and S. Yankielowicz, *Supergravity solutions for branes localized within branes*, *Phys. Lett.* **B 432** (1998) 298 [`hep-th/9803103`].

[6] A. Hashimoto, *Supergravity solutions for localized intersections of branes*, *J. High Energy Phys.* **01** (1999) 018 [`hep-th/9812159`].

JHEP09(2001)018

[7] A. Gomberoff and D. Marolf, *Brane transmutation in supergravity*, *J. High Energy Phys.* **02** (2000) 021 [hep-th/9912184].

[8] D. Marolf and A. Peet, *Brane baldness vs. superselection sectors*, *Phys. Rev.* **D 60** (1999) 105007 [hep-th/9903213].

[9] A. Fayyazuddin and D.J. Smith, *Localized intersections of M5-branes and four-dimensional superconformal field theories*, *J. High Energy Phys.* **04** (1999) 030 [hep-th/9902210].

[10] A. Fayyazuddin and D.J. Smith, *Warped AdS near-horizon geometry of completely localized intersections of M5-branes*, *J. High Energy Phys.* **10** (2000) 023 [hep-th/0006060].

[11] R. Kallosh and J. Kumar, *Supersymmetry enhancement of Dp-branes and M-branes*, *Phys. Rev.* **D 56** (1997) 4934 [hep-th/9704189].

[12] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, *Phys. Rev.* **D 61** (2000) 086001 [hep-th/9911161].

[13] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

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